Supplementary Appendix to “Formal Models of Nondemocratic Politics”

Scott Gehlbach*

Konstantin Sonin†

Milan W. Svolik‡

*Department of Political Science, University of Wisconsin–Madison, Madison, Wisconsin 53706; email: gehlbach@polisci.wisc.edu.

†Harris School of Public Policy Studies, University of Chicago, Chicago, Illinois 60637, and Higher School of Economics, Moscow, 101000, Russia; email: ksonin@uchicago.edu.

‡Department of Political Science, Yale University, New Haven, Connecticut 06511; email: milan.svolik@yale.edu.
The probability that a member of the winning coalition will be among the $W$ members of the selectorate with the highest realization of the affinity parameters $A_i$: The probability that an observation $x$ from a sample of $N$ draws from the standard uniform density is the $k$th largest is

$$\binom{N - 1}{k - 1} x^{(N-1)-(k-1)} (1 - x)^{k-1}. $$

If all we know about $x$ is that it is drawn from the standard uniform density, then the probability that $x$ will rank as the $k$th largest observation is

$$\int_0^1 \binom{N - 1}{k - 1} x^{(N-1)-(k-1)} (1 - x)^{k-1} \, dx. $$

In turn, the probability that this observation will be at least the $k$th largest observation is

$$\sum_{i=1}^{k} \int_0^1 \binom{N - 1}{i - 1} x^{(N-1)-(i-1)} (1 - x)^{i-1} \, dx. $$

After multiplying and dividing the expression through by $N$, the integrand in each element of this sum can be expressed as the density function of the Beta distribution with the parameters $N - i + 1$ and $i$

$$\sum_{i=1}^{k} \frac{1}{N} \int_0^1 \frac{\Gamma(N + 1)}{\Gamma(i)\Gamma(N - i + 1)} x^{(N-i)} (1 - x)^{i-1} \, dx. $$

Since this density function (by assumption) integrates to 1, the probability that an observation from a sample of $N$ draws from the standard uniform density is at least the $k$th largest is

$$\sum_{i=1}^{k} \frac{1}{N} = \frac{k}{N}. $$
Thus we see that the probability that a member of the winning coalition of size $W$ who considers defecting to the challenger expects to be among the $W$ members of the selectorate of size $S$ with the highest realization of the affinity parameters $A^i_C$ with the probability $\frac{W}{S}$.