There are four problems, each with multiple parts. Please be sure you understand what a problem is asking before beginning to work on it—I will give little credit for the correct answer to the wrong problem. Clearly indicate your final answer (e.g., by circling it) to each part of each problem. Further, do show enough of your work that I can give partial credit if necessary, but given that constraint please be as concise as possible.

Three of the four problems on the exam are worth 24 points, whereas one of the four problems is worth 12 points. Which of the four problems is worth 12 points is your decision. Please clearly indicate your choice.
1. (Veto players) This problem explores the argument in Section 4.2 that the addition of veto players may make policy change *more* likely when the new veto player acquires agenda-setting power. Assume three actors $A, B, C$. The following figure, which depicts the configuration of ideal points for the three veto players, illustrates the equilibrium policy when only $A$ and $B$ are veto players and $B$ is the agenda setter.

(a) On the same figure, plot the equilibrium policy when $A$, $B$, and $C$ are veto players and $C$ is the agenda setter.

(b) For what values of the status quo is policy change *greater* when $C$ is a veto player (and has agenda-setter power) than when it is not (and $B$ has agenda-setting power)? Interpret your result.

2. (Legislative bargaining) Consider the following legislative ultimatum game. There is a legislature of $N \geq 3$ members ($N$ odd), indexed by $i$, one of whom is chosen at random to be agenda setter. Whoever is chosen to be agenda setter proposes a vector of district-specific benefits $\mathbf{x} = (x_1, \ldots, x_N)$. In a departure from the model in Section 6.1.1, we assume that this distribution is financed with a tax assessed on all members (i.e., as opposed to being a share of an exogenously defined resource). In particular, denoting by $x = \sum_i x_i$ the total benefits paid to all members, assume that the cost of the tax to any individual member is $\frac{\kappa x^2}{N}$. Thus, the payoff to any member $i$ from a distribution $\mathbf{x}$ is

$$x_i - \frac{\kappa x^2}{N}.$$  

(1)

Assume that the default policy $\bar{\mathbf{x}} = (0, \ldots, 0)$ and that members play weakly undominated voting strategies.
(a) As a function of $x$, what benefit $x_i$ must any legislator $i$ receive to vote for a proposal $x$? (Do not make this part of the problem harder than it is. The answer is in Equation 1.)

(b) Using your answer to the previous part of the problem, write down the agenda setter’s payoff as a function of $x$. (Hint: For a given $x$, the agenda setter receives $x - \sum_{j \in C} x_j$, where $C$ is the set of $\frac{N-1}{2}$ members chosen to be in the minimum winning coalition, and like all other members, the agenda setter pays the tax assessed on all members.)

(c) Solve for the $x$ optimally chosen by the agenda setter.

(d) Using your answers to parts (a) and (c) of the problem, solve for the benefit $x_i$ optimally provided to any member $i$ of the winning coalition, other than the agenda setter herself.

(e) Using your answers to parts (b) and (d) of the problem, find the agenda setter’s equilibrium payoff. How does this depend on parameters of the model?

3. (Political agency) Consider the following extension of the signaling model of political agency in Section 7.3. Following choice of $e_1$, but before the election, voters observe a perfectly informative signal of the incumbent’s type (diligent or lazy) with probability $\alpha$, whereas with probability $1 - \alpha$ they receive a completely uninformative signal. Thus, with probability $\alpha$ voters “know” the incumbent’s type, whereas with probability $1 - \alpha$ they must infer it (as in Section 7.3) from the incumbent’s effort choice. The model in Section 7.3 is a special case of the model here, with $\alpha = 0$. One interpretation is that $\alpha$ measures media quality, where a high $\alpha$ means that the media are more likely to obtain verifiable information about the incumbent’s type.

Observe that the second period is identical to that in Section 7.3. Thus, as before, voters prefer to reelect an incumbent whom they believe to be diligent with probability greater than $\pi$, which is the probability that the challenger is diligent. We look for an equilibrium in which voters reelect the incumbent if and only if $e_1 = 1$.

(a) What is the probability that a lazy incumbent is reelected if she chooses $e_1 = 0$? What is her expected payoff from doing so? (Hint: The answer to this part of the problem does not depend on $\alpha$.)

(b) What is the probability that a lazy incumbent is reelected if she chooses $e_1 = 1$? What is her expected payoff from doing so?

(c) With what probability does a lazy incumbent choose $e_1 = 1$?

(d) How does the incentive for a lazy incumbent to choose $e_1 = 1$ depend on $\alpha$? Interpret your result.

(e) How does voter welfare depend on $\alpha$? (I am looking for a qualitative answer, not necessarily a calculation.)

4. (Markov games) Consider the following Markov game. There are two states: $PD$ and $ABC$. Action spaces and payoffs are given by the following matrixes, where the matrix
on the left corresponds to the state $PD$ and the matrix on the right corresponds to the state $ABC$:

$$
\begin{array}{ccc}
C & D \\
C & 0,0 & -2,1 \\
D & 1,-2 & -1,-1 \\
\end{array}
\quad
\begin{array}{ccc}
A & B & C \\
A & 4,2 & -1,-1 & -1,-1 \\
B & -1,-1 & 2,4 & -1,-1 \\
C & -1,-1 & -1,-1 & 5,5 \\
\end{array}
$$

The game begins in the state $PD$. In any period that the state is $PD$, if both players play $C$, then the game transitions to the state $PD$ (i.e., remains in the same state the following period) with probability $q$ and transition to the state $ABC$ with probability $1 - q$. In any period that the state is $PD$, if any player defects, then the game transitions to the state $PD$ the following period with certainty. The state $ABC$ is an absorbing state.

We derive the condition for existence of a stationary equilibrium in which both players choose $C$ when the state is $PD$ and $C$ when the state is $ABC$.

(a) Derive $V_1(ABC, (C, C)) = V_2(ABC, (C, C))$, the value to players 1 and 2 of being in the state $ABC$ when the players always choose $(C, C)$ in that state. Demonstrate that no player has an incentive to deviate from her equilibrium strategy in this state.

(b) Derive $V_1(PD, (C, C)) = V_2(PD, (C, C))$, the value to players 1 and 2 of being in the state $PD$ when the players always choose $(C, C)$ in that state. Use your answer to the previous part of the problem to simply the expression.

(c) Use the one-deviation property to derive the condition such that no player has an incentive to deviate from her equilibrium strategy in the state $PD$.

(d) If you have time, derive the condition for existence of a stationary equilibrium in which both players choose $C$ when the state is $PD$ and $A$ when the state is $ABC$. 

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