Political Science 836—Spring 2015
Midterm Exam

There are three problems, each with multiple parts, worth 45 points total. Please be sure you understand what a problem is asking before beginning to work on it—I will give little credit for the correct answer to the wrong problem. Clearly indicate your final answer (e.g., by circling it) to each part of each problem. Further, do show enough of your work that I can give partial credit if necessary, but given that constraint please be as concise as possible.

1. (15 points—Electoral competition with endogenous entry) Consider the following model of electoral competition with endogenous entry. There is a continuum of voters, where any voter i has preferences over policy $x \in [0, 1]$ represented by the utility function

$$u_i(x) = -|x - x_i|.$$

Voters' ideal points are distributed uniformly on $[0, 1]$.

In contrast to citizen-candidate models in the tradition of Osborne and Slivinski (1996) and Besley and Coate (1997), not all voters are potential candidates. Rather, there are only eight potential candidates: two with ideal point 0, two with ideal point $\frac{1}{3}$, two with ideal point $\frac{2}{3}$, and two with ideal point 1. Potential candidates suffer policy loss as do voters (i.e., they have absolute-value preferences). In addition, any potential candidate incurs a cost $\delta > 0$ if she enters the race and a payoff $v \geq 0$ if she wins.

The timing of events is as follows: All potential candidates simultaneously and independently decide whether to enter the race. Following this, voters decide for whom to vote. Assume that voters vote sincerely for the candidate whose position (ideal point) they most prefer, employing an equal-probability rule if indifferent.

(a) Derive the condition(s) for existence of a two-candidate equilibrium in which one candidate has ideal point $\frac{1}{3}$ and one has ideal point $\frac{2}{3}$. If such an equilibrium does not exist for any values of δ and v, explain why.

(b) Derive the condition(s) for existence of a two-candidate equilibrium in which one candidate has ideal point 0 and one has ideal point 1. If such an equilibrium does not exist for any values of δ and v, explain why.

Now assume that the set of voters is finite but large, with ideal points distributed uniformly (i.e., at identical intervals) on $[0, 1]$. Assume that the number of voters is even, such that there is no voter with ideal point $\frac{1}{2}$. Voters vote strategically; restrict attention to equilibria in which voters play weakly undominated voting strategies. Potential candidates do not vote.

(c) Specify strategies off the equilibrium path, in which a third candidate has entered, such that the equilibrium in part (b) exists for a larger region of the parameter space than you found there. Provide the condition(s) for existence of the equilibrium you define.

2. (15 points—Electoral competition under uncertainty) Consider the following variant on the Wittman model with uncertainty presented in Section 2.2. Parties have preferences as before. Voters have Euclidean preferences over $x \in \Re$, with ideal points distributed uniformly on the interval $[\eta, 1 + \eta]$, where η is a random variable, realized after parties choose positions, distributed uniformly on the interval $[-\beta, \beta]$, where $\beta < \frac{1}{2}$. The parameter β thus measures the degree of uncertainty about the distribution of voters' preferences. (You may find it helpful to draw a picture of the distribution of voters' preferences. This should be a rectangle, the location of which depends on the realization of the random variable η.)

Solve for the equilibrium of this game as follows. Assume preliminarily that $0 < x_L < x_R < 1$ and $\frac{x_L + x_R}{2} \in [\eta, 1 + \eta]$ for all possible realizations of η. Then:

(a) Holding η constant, derive the share of voters who support party L.

(b) Use the distribution of η to derive the probability that L wins as a function of parameters of the model.

(c) Use this expression to solve for the optimal policies by L and R.

(d) Verify that the preliminary assumptions given above hold in equilibrium.

3. (15 points—Various models of policy choice) Consider the following environment. There is a continuum of citizens, of whom α are rich (r) and $1 - \alpha$ are poor (p). Assume $\alpha \in \left(0, \frac{1}{2}\right)$, so that the median citizen is poor. Total income in the economy is normalized to one. The rich receive proportion θ of this income, where $\theta > \alpha$, so that the poor receive proportion $1 - \theta$. (We interpret θ as the level of income inequality in the economy.) Thus, the per-capita income of any rich citizen is $y_r = \frac{\theta}{\alpha}$, whereas the per-capita income of any poor citizen is $y_p = \frac{1-\theta}{1-\alpha}$. Average income is

$$\bar{y} = \alpha y_r + (1 - \alpha)\, y_p = 1.$$

At issue is the tax rate $\tau \in [0, 1]$. Proceeds from taxation fund public goods enjoyed equally by rich and poor citizens. Assume in particular that the utility of any rich citizen is

$$u_r(\tau) = y_r(1 - \tau) + \beta \ln(\tau \bar{y}) = \frac{\theta(1 - \tau)}{\alpha} + \beta \ln(\tau),$$

whereas the utility of any poor citizen is

$$u_p(\tau) = y_p(1 - \tau) + \beta \ln(\tau \bar{y}) = \frac{(1 - \theta)(1 - \tau)}{1 - \alpha} + \beta \ln(\tau).$$

Thus, the utility for any citizen is given by the sum of post-tax income and a payoff from public-goods provision, which is funded by tax receipts $\tau \bar{y}$. Assume $\beta \in (0, 1)$, which ensures that the tax rate is bounded by zero and one in all parts of the problem.

(a) Derive the most-preferred policy for rich and poor, respectively.

(b) Find the policy that would be chosen by a Benthamite social planner, that is, the utilitarian outcome. (Observe that this is the policy that would be chosen through electoral competition in the environment of Section 2.1.1 if we assume that preference heterogeneity is identical across groups.)

(c) Hotelling-Downs: There are two political parties $P = A, B$, each of which is office-seeking. Each party P announces a tax rate $\tau_P \in [0, 1]$. Citizens vote for the party whose announced tax rate maximizes their utility. What is the tax rate chosen by each party in equilibrium?

(d) Lobbying: There is an elected politician who represents all citizens but who chooses the tax rate $\tau \in [0,1]$ under the influence of a single lobby representing the rich. In particular, the politician maximizes a weighted average of the aggregate utility of all citizens and contribution paid by the lobby:

$$\gamma\left[\alpha u_r(\tau) + (1-\alpha)\, u_p(\tau)\right] + C.$$

The parameter $\gamma > 0$ measures the degree to which the politician values the welfare of all citizens versus contributions by the rich. The lobby maximizes the aggregate post-tax income of all rich citizens, net of any contribution paid to the politician:

$$\alpha u_r(\tau) - C.$$

What is the tax rate chosen by the politician in equilibrium?

(e) Plot the policies found in parts (a)–(d) on a line. How does the difference among these policies depend on income inequality, as measured by the parameter θ.