Political Science 836—Fall 2013
Midterm Exam

There are four problems, each with multiple parts, worth 60 points total. Please be sure you understand what a problem is asking before beginning to work on it—I will give little credit for the correct answer to the wrong problem. Clearly indicate your final answer (e.g., by circling it) to each part of each problem. Further, do show enough of your work that I can give partial credit if necessary, but given that constraint please be as concise as possible.

1. (15 points—Electoral competition under certainty) Consider the following model of electoral competition. There are two parties $P = A, B$, where each party names a tax rate $t_P \in [0, 1]$ to attract votes from a continuum of voters who vote for the party whose position they most prefer. In particular, any voter i receives utility from tax rate t of

$$u_i(t) = (1 - t)y_i + t\bar{y},$$

where y_i is voter i's income and $\bar{y}$ is mean income. Thus, tax revenue is returned as a lump-sum transfer to all voters, with no deadweight loss from taxation. Assume that the distribution of voters' income is continuous and strictly increasing, with median income $y_m < \bar{y}$. Thus, as in any society, mean income is greater than median income.

   (a) Derive the optimal tax rate for a generic citizen i with income y_i. (Hint: When is the derivative of $u_i(t)$ with respect to t positive/negative?) What is t_m, the median most-preferred tax rate?

   (b) Office-motivated parties: Find all pure-strategy Nash equilibria of the game in which parties care only about maximizing their probability of winning.

   (c) Policy-motivated parties: Label the parties $P = A, B$, and assume that each party P derives utility from tax rate t of

$$u_P(t) = (1 - t)y_P + t\bar{y}$$

   where $y_A < \bar{y} < y_B$. Thus, party A is the party of the "poor," whereas party B is the party of the "rich." Show that it is a Nash equilibrium for each party to name t_m, the median most-preferred tax rate. Is this the unique Nash equilibrium?

2. (15 points—Citizen candidates) Consider the citizen-candidate model in Section 1.4.3, only now assume that voting is by the runoff rule (as in Exercise 1.1) rather than by plurality rule. Assume further that citizens vote sincerely for the candidate whose position they most prefer, abstaining if indifferent among all candidates who have entered. Finally, assume that the payoff from holding office $v > \delta$.

   (a) Derive the condition for existence of an equilibrium in which two citizens with median ideal point x_m enter as candidates, as follows:

      i. Verify that no citizen who has entered has an incentive to deviate by not entering, or derive the condition such that this is the case.

      ii. Verify that no citizen with median ideal point x_m who has not entered has an incentive to deviate by entering, or derive the condition such that this is the case.

      iii. Verify that no citizen with ideal point $x_i \neq x_m$ who has not entered has an incentive to deviate by entering, or derive the condition such that this is the case.

      iv. Write any conditions derived in the previous three parts of the problem as a single condition. Interpret your result.

(b) How does your answer to part (a) compare to that with plurality voting? (Recall that the text considers two-candidate equilibria when $v = 0$, which is not the case examined here.)

(c) Derive the condition for existence of an equilibrium with k candidates with ideal point x_m, where $k \geq 2$.

3. Omitted

4. (15 points—Various models of policy choice) Consider the following environment. There are two groups of citizens, L and R, with preferences over policy $x \in \Re$ as follows:

$$u_L(x) = -(x - x_L)^2$$
$$u_R(x) = -(x - x_R)^2,$$

where $x_L < x_R$. There are α_L voters in group L and α_R voters in group R, with $\alpha_R > \alpha_L$.

The following problems explore political conflict between left and right using three alternative models of policy choice.

(a) Hotelling-Downs: There are two political parties $P = A, B$, each of which is office-seeking. Each party P announces a policy $x_P \in \Re$. Citizens vote for the party whose announced policy maximizes their utility. What is the policy chosen by each party in equilibrium?

(b) Individual uncertainty: There are two political parties $P = A, B$, each of which is office-seeking. Each party P announces a policy $x_P \in \Re$. Citizens vote as in the model of Section 2.1.1, where we assume that preference heterogeneity is identical within each group (i.e., $\omega_L = \omega_R = \omega$). What is the policy chosen by each party in equilibrium?

(c) Lobbying: There is an elected politician who represents the right but who chooses policy $x \in \Re$ under the influence of a single lobby representing the left. In particular, the politician maximizes a weighted average of the aggregate utility of all right citizens and any contribution paid by the lobby:

$$-\gamma\alpha_R (x - x_R)^2 + C.$$

The parameter $\gamma \in (0, 1)$ measures the degree to which the politician values the welfare of right citizens versus contributions by the left. The lobby maximizes the aggregate post-tax income of all left citizens, net of any contribution paid to the politician:

$$-\alpha_L (x - x_L)^2 - C.$$

What is the policy chosen by the politician in equilibrium?

(d) Find the policy x that would be chosen by a Benthamite social planner.

(e) Plot the four policies found in parts (a)–(d) on a line. What explains the ordering of these policies?