Political Science 836—Fall 2016
Midterm Exam

There are three problems, each with multiple parts, worth 45 points total. Please be sure you understand what a problem is asking before beginning to work on it—I will give little credit for the correct answer to the wrong problem. Clearly indicate your final answer (e.g., by circling it) to each part of each problem. Further, please show enough of your work that I can give partial credit if necessary, but given that constraint please be as concise as possible.

You have until 3:15 to finish the exam. When you are finished, please give your exam to Deb McFarlane in the department office on the first floor. Deb will expect your exam by 3:20.

Good luck!

1. (15 points—Hotelling-Downs competition under certainty and uncertainty) Consider Hotelling-Downs competition in the following environment. Policy takes one of two values, $x \in \{0, 1\}$. Two parties, $P = A, B$, compete by adopting positions in this policy space. Proportion α of citizens most prefer $x = 0$, where $\alpha \in \left(\frac{1}{2}, 1\right)$, whereas proportion $1-\alpha$ most prefer $x = 1$. The median citizen is therefore a "0" type. Voters care not just about policy, however, but also about characteristics of the parties over which the parties themselves have no control. In particular, type-0 citizens vote for party A if

$$-(x_A - 0)^2 > -(x_B - 0)^2 + \theta,$$

where $\theta > 0$ is a parameter of the model (i.e., not a random variable); vote for party B if the inequality is reversed; and abstain otherwise. Type-1 citizens, in contrast, vote for party A if

$$-(x_A - 1)^2 > -(x_B - 1)^2 + \rho,$$

where $\rho < 0$; vote for party B if the inequality is reversed; and abstain otherwise.

   (a) Assume $\theta > 1$. What is the set of pure-strategy Nash equilibria?

   (b) Assume $\theta \in (0, 1)$. What is the set of pure-strategy Nash equilibria?

   Now consider a related model, again with office-seeking parties. Voting is as in Section 2.1.1 (individual but not aggregate uncertainty), but for the following assumptions: There are two groups of voters, $g = 0, 1$, where proportion α of voters are in group 0. Voters in group 0 vote for party A if

$$-(x_A - 0)^2 > -(x_B - 0)^2 + \theta + \eta_{i0},$$

   where $\theta > 0$, and for party B otherwise. Similarly, voters in group 1 vote for party A if

$$-(x_A - 1)^2 > -(x_B - 1)^2 + \rho + \eta_{i1},$$

   where $\rho < 0$, and for party B otherwise. For either group g, η_{ig} is distributed uniformly on the interval $\left[-\frac{1}{2\omega}, \frac{1}{2\omega}\right]$. Assume that θ and ρ are "small," in the sense that the share of voters in either group lies strictly between zero and one for all platforms (x_A, x_B).

   (c) What is the equilibrium strategy profile (x_A^*, x_B^*)? (I will give partial credit for the correct intuition. Please use your time wisely.)

2. (15 points—Citizen candidates) Consider the following citizen-candidate model. There is a continuum of citizens of mass one, partitioned into three groups of equal size. The winning candidate chooses a division (q_1, q_2, q_3) of a "pie," where q_g is the per-capita share of the pie received by citizens in group g. Assume that the pie is of size one-third, so that distributing the entire pie to, say, group 1 implies a division $(1, 0, 0)$. In the event no candidate has entered the race, a status-quo division $\bar{q} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is implemented. Citizens receive utility equal to their per-capita share of the pie, net of the cost of entry $\delta > 0$. There is no exogenous payoff from winning (i.e., $v = 0$). Citizens vote sincerely, abstaining if indifferent among candidates and choosing according to an equal-probability rule if indifferent between two candidates they prefer to a third.

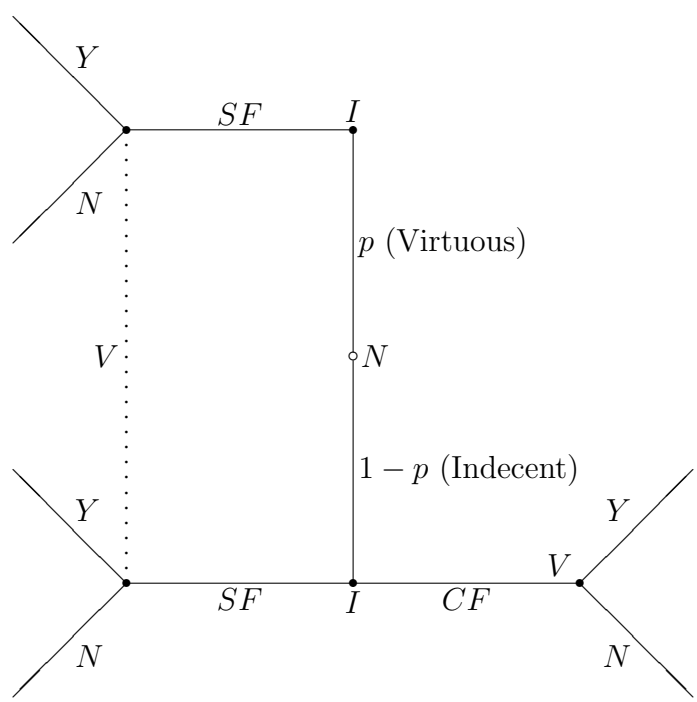
(a) Derive the condition for existence of a one-candidate equilibrium. If no such equilibrium exists for any value of δ, explain why.

(b) Derive the condition for existence of a two-candidate equilibrium in which two citizens from the same group enter. If no such equilibrium exists for any value of δ, explain why.

(c) Derive the condition for existence of a two-candidate equilibrium in which the candidates represent distinct groups (e.g., one candidate from group 1 and one candidate from group 2). If no such equilibrium exists for any value of δ, explain why.

3. (15 points—Informative campaign finance) This problem illustrates the logic of an argument made during last Sunday's presidential debate. You need not have seen the debate to understand the model, which takes no position on the empirical validity of the argument.

Consider a variant of the model of informative campaign finance in Section 3.3, as summarized in the following figure:



The incumbent is one of two types, virtuous or indecent. The voter's prior belief is that the incumbent is virtuous with probability $p \in \left(\frac{1}{2}, 1\right)$ and that the challenger is virtuous with probability one-half. A virtuous type always self-finances (SF) his campaign. In contrast, an indecent type may either self-finance (SF) his campaign or raise campaign funds from special interests (CF). The indecent type makes this choice to maximize the expected payoff from holding office, net of the cost of self-finance:

$$\Pr(\text{win} \mid a) \cdot w - a\psi,$$

where $a = 1$ indicates self-financing and $a = 0$ indicates raising campaign funds from special interests, $w > 0$ is the exogenous payoff from reelection, and $\psi > 0$ is the cost of self-financing the campaign.

To derive the probability that the incumbent wins, assume that a representative voter reelects the incumbent if

$$\mu(a)\,\theta \geq \frac{1}{2}\theta + \epsilon,$$

where $\mu(a)$ is the posterior belief that the incumbent is virtuous, given a; θ is the differential payoff from electing a virtuous type; and ϵ is a random variable drawn from the uniform distribution with support $[-\sigma, \sigma]$. Thus, the probability that the incumbent wins is

$$\frac{1}{2} + \frac{1}{2\sigma}\left(\mu(a) - \frac{1}{2}\right)\theta.$$

(a) Derive the condition for existence of a separating equilibrium in which the virtuous type self-finances and the indecent type raises campaign funds from special interests. Interpret your result.

(b) Derive the condition for existence of a pooling equilibrium in which both types self-finance. Interpret your result.