Political Science 836—Fall 2017
Final Exam

There are four problems, each with multiple parts, worth 70 points total. Please be sure you understand what a problem is asking before beginning to work on it—I will give little credit for the correct answer to the wrong problem. Clearly indicate your final answer (e.g., by circling it) to each part of each problem. Further, please show enough of your work that I can give partial credit if necessary, but given that constraint please be as concise as possible.

You have until 1:00 to finish the exam. Good luck!

1. (20 points—Veto players) This problem follows Dragu, Fen, and Kuklinski (2014) in modeling constitutional review as a veto-players game. Consider a model with two players: a government (G) and a court (C). The government and court have Euclidean preferences over policy $x \in [0, \infty)$, with ideal points x_G and x_C, respectively. (Below we compare the case in which these preferences are common knowledge and that in which they are not.) The government chooses a policy $p \in [0, \infty)$, which the court can either accept or reject. If the court accepts the policy, the outcome $x = p$; otherwise, $x = 0$ (i.e., the status quo policy is 0).

   (a) Assume that x_G and x_C are common knowledge, with $0 < x_C < x_G$. (For this and subsequent parts of the problem, it may be useful to draw a picture.) Derive the equilibrium policy as follows:

      i. What policies does the court prefer to the status quo and therefore accept?

      ii. What policy p does the government optimally choose, given the anticipated action of the court?

      iii. Is the equilibrium outcome (p or 0, depending on the action of the court) Pareto efficient—that is, there is no other outcome such that both the government and court are at least as well off and at least one actor is strictly better off?

   (b) Now assume that x_C is private knowledge of the court, where the government's prior belief is that x_C is distributed uniformly on the interval $[0, \kappa]$, with $\kappa < x_G < 2\kappa$. Derive the equilibrium policy as follows:

      i. As a function of x_C, what policies does the court prefer to the status quo and therefore accept?

      ii. From the perspective of the government, which does not observe x_C, what is the probability that the court accepts some policy $p \in [0, x_G]$?

      iii. Derive the government's optimal policy choice, which solves

$$\max_p \Pr(\text{accept} \mid p)\,(p - x_G) + [1 - \Pr(\text{accept} \mid p)]\,(-x_G)\,.$$

      How does the optimal policy relate to parameters of the model?

      iv. For all realizations of x_C, is the equilibrium outcome Pareto efficient?

2. (20 points—Legislative bargaining) Consider the following infinite-horizon legislative bargaining environment. There are three legislators, indexed by i, who have an equal probability of recognition and common discount factor δ. Rather than bargaining over distribution of a resource, as in the Baron-Ferejohn model, we assume that legislators bargain over policy $x \in [0, 1]$, where the status-quo policy $\bar{x} = 1$. Legislators have preferences over policy given by:

$$u_1(x) = u_2(x) = 1 - x,$$
$$u_3(x) = x.$$

Each legislator receives her payoff from the status-quo policy until some alternative is agreed to, after which she receives her payoff from the alternative in perpetuity. Thus, for example, if no agreement is reached in the first period, and a proposal of $\hat{x}$ is agreed to in the second period, then legislator 3 receives a discounted payoff of $1+\frac{\delta}{1-\delta}\hat{x}$, where we use the status-quo $\bar{x} = 1$.

In contrast to the Baron-Ferejohn model, in which there exist distributions that improve on the status quo for all legislators, any change from the status quo in this game leaves legislator 3 strictly worse off. This implies that bargaining may continue beyond the first period. To see this, we construct a stationary equilibrium in which:

- if legislator 1 or 2 is chosen as agenda setter, that legislator proposes $x = 0$,
- if legislator 3 is chosen as agenda setter, that legislator proposes $x = 1$,
- legislators 1 and 2 vote to accept any proposal $x \leq x^*$, and
- legislator 3 votes to accept any proposal $x \geq x^*$,

where $x^* \in (0, 1)$ is a value to be derived below. As constructed, bargaining continues until legislator 1 or 2 is recognized. Complete the following steps to derive the threshold x^* and show that this is an equilibrium.

(a) For legislators 1 and 2, derive $V_1 = V_2$, the expected payoff in equilibrium at the beginning of any period *in which a proposal has not been accepted.*

(b) For legislator 3, derive V_3, the expected payoff in equilibrium at the beginning of any period *in which a proposal has not been accepted.*

(c) Using your answer to part (a), derive the x^* such that legislators 1 and 2 are just indifferent between accepting $x = x^*$, which gives a discounted payoff of $\frac{1-x^*}{1-\delta}$, and rejecting that proposal. (If indifferent between accepting and rejecting $x = x^*$, legislators 1 and 2 strictly prefer to accept any $x < x^*$ and reject any $x > x^*$.)

(d) Using your answer to part (b) and the value of x^* derived in part (c), show that legislator 3 is indifferent between accepting $x = x^*$ and rejecting that proposal. (If indifferent between accepting and rejecting $x = x^*$, legislator 3 strictly prefers to accept any $x > x^*$ and reject any $x < x^*$.)

(e) How does the threshold x^* depend on the discount factor δ? Interpret your result.

3. (20 points—Markov games) Consider the following Markov game. There are two states: *Blue* and *Green*. Action spaces and payoffs are given by the following matrixes, where the matrix on the left corresponds to the state *Blue* and the matrix on the right corresponds to the state *Green*:

	A	B	C
A	1,1	0,0	0,0
B	2,0	0,0	0,0
C	3,0	0,0	0,0

	L	R
L	4,4	0,0
R	0,0	4,4

The game begins in the state *Blue*. In any period that the state is *Blue*, if both players play A, then the game transitions to the state *Blue* (i.e., remains in the same state the following period) with probability q and transitions to the state *Green* with probability $1 - q$. In any period that the state is *Blue*, if any player defects, then the game transitions to the state *Blue* the following period with certainty. The state *Green* is an absorbing state.

Derive the condition(s) for existence of a Markov perfect equilibrium in which both players play A when the state is *Blue* and L when the state is *Green*, as follows.

(a) Derive $V_1(Green, (L, L)) = V_2(Green, (L, L))$, the value to players 1 and 2 of being in the state *Green* when the players always play (L, L) in that state. Demonstrate that no player has an incentive to deviate from her equilibrium strategy in this state.

(b) Derive $V_1(Blue, (A, A)) = V_2(Blue, (A, A))$, the value to players 1 and 2 of being in the state *Blue* when the players always play (A, A) in that state. Use your answer to the previous part of the problem to simply the expression.

(c) What condition, if any, must hold for Player 1 to have no incentive to deviate in the state *Blue*?

(d) What condition, if any, must hold for Player 2 to have no incentive to deviate in the state *Blue*?

4. (10 points—Miscellaneous) If you have extra time, you may occupy yourself answering the following questions. Please note that each question is worth only $3\frac{1}{3}$ points—you probably do not want to turn attention to this problem unless you are confident of your answers to earlier problems.

(a) The model in problem 1 is a stylized representation of the U.S. system, in which (with some exceptions) constitutional review can be initiated only after implementation of some policy. Dragu, Fen, and Kuklinski refer to this as *ex post review*. How, if at all, would you expect the equilibrium policy to be different under a system of *ex ante review*, in which the court first names a legal limit l, such that any policy $p > l$ will be vetoed, following which the government chooses a policy $p \in [0, \infty)$?

(b) Consider a generalization of problem 2 in which there are $N \geq 2$ legislators with most-preferred policy $x = 0$, though as above only one legislator with most-preferred policy $x = 1$. How does the threshold x^* depend on N? Interpret your result.

(c) In problem 3, what condition(s) must hold for there to exist a Markov perfect equilibrium in which both players play B when the state is *Blue* and L when the state is *Green*?