There are four problems, each with multiple parts, worth 52.5 points total. Please be sure you understand what a problem is asking before beginning to work on it—I will give little credit for the correct answer to the wrong problem. Clearly indicate your final answer (e.g., by circling it) to each part of each problem. Further, please show enough of your work that I can give partial credit if necessary, but given that constraint please be as concise as possible.

You have until 11:45 to finish the exam. Good luck!
1. (10 points—Electoral competition) Consider the following $2 \times 2$ table:

<table>
<thead>
<tr>
<th></th>
<th>Exogenous candidates</th>
<th>Endogenous candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position-taking</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Not position-taking</td>
<td>(3)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

We have discussed various models of electoral competition that can be assigned to some cell of this table, where for each model the set of candidates (parties) is either exogenous or endogenous and the candidates (parties) either do or do not compete by explicitly adopting (credible) campaign platforms. The Hotelling-Downs model, for example, is a model of exogenous candidates (in its most basic formulation, two parties, $P = A, B$) in which each candidate (party) takes a position, implicitly committing to implement that policy if elected.

(a) Populate this table with the various models we have discussed. Thus, for example, you might begin with “(1) Hotelling-Downs model, . . .”

(b) Briefly discuss the key theoretical insights from each of the models you list in (a).

2. (20 points—Various models of policy choice) Consider the model of Problem Set 3 in which two groups, the poor ($p$) and rich ($r$), have preferences over tax policy $\tau \in [0, 1]$ characterized by *self-centered inequity aversion*, as defined by Fehr and Schmidt (1999). In this model, any poor citizen has preferences represented by the payoff function

$$u_p(\tau) = (1 - \tau)y_p + \left(\tau - \frac{\gamma}{2}\tau^2\right)(\alpha_py_p + \alpha_ry_r) - \kappa_p (1 - \tau)(y_r - y_p),$$

where $\kappa_p > 0$, whereas any rich citizen has preferences represented by

$$u_r(\tau) = (1 - \tau)y_r + \left(\tau - \frac{\gamma}{2}\tau^2\right)(\alpha_py_p + \alpha_ry_r) - \kappa_r (1 - \tau)(y_r - y_p),$$

where $\kappa_r > 0$. As before, we assume that the rich are wealthier than the poor, that is, $y_r > y_p$. We additionally assume that the poor are more numerous than the rich: $\alpha_p > \alpha_r$. In what follows, you may find it convenient to define average income $\bar{y} \equiv \alpha_py_p + \alpha_ry_r; $ clearly, $y_p < \bar{y} < y_r$.

(a) Derive the tax rate most preferred by the poor and the rich, respectively.

(b) What is the tax rate that would be chosen by a Benthamite social planner, that is, the utilitarian outcome?

(c) Hotelling-Downs: There are two political parties $P = A, B$, each of which is office-seeking. Each party $P$ announces a tax rate $\tau_P \in [0, 1]$. Citizens vote for the party whose announced tax rate maximizes their utility. What is the tax rate chosen by each party in equilibrium?

(d) Lobbying: There is an elected politician who represents all citizens but who chooses the tax rate $\tau \in [0, 1]$ under the influence of a single lobby representing the rich. In particular, the politician maximizes a weighted average of the aggregate utility of all citizens and contribution paid by the lobby:

$$\chi \left[\alpha_p u_p(\tau) + \alpha_r u_r(\tau)\right] + C.$$
The parameter $\chi > 0$ measures the degree to which the politician values the welfare of all citizens versus contributions by the rich. The lobby maximizes the aggregate post-tax income of all rich citizens, net of any contribution paid to the politician:

$$\alpha_r u_r (\tau) - C.$$ 

What is the tax rate chosen by the politician in equilibrium?

(e) How do the tax rates identified in (a)–(d) compare to each other? If the relationship between any two tax rates is ambiguous, please explain why.

3. (15 points—Informative campaign finance) Consider the following adaptation of the model of informative campaign finance of Section 3.3. There are two strategic actors: an incumbent and a voter. The voter’s prior belief is that the incumbent is “good” with probability $p$ and “bad” with probability $1 - p$, whereas the voter’s prior belief is that the challenger is “good” with probability $q$ and “bad” with probability $1 - q$. As before, the marginal benefit to the voter of electing a good type is $\theta > 0$.

We assume that the incumbent knows whether the challenger is bad (e.g., because of opposition research) and is able to communicate this fact through advertising. In contrast, the incumbent is unable to credibly communicate her own type. As before, the challenger cannot advertise. The following figure illustrates the timing of events, where “good” and “bad” refer to the challenger’s type (known to the incumbent) and $Y$ and $N$ refer to the voter’s decision to reelect the incumbent or not, respectively:

As before, the voter suffers a cost of $\phi > 0$ if the incumbent advertises and is elected. All other elements of the game are identical to the model in Section 3.3.
Solve for the condition on parameters such that there is a *separating* equilibrium in which the incumbent advertises if she knows that the challenger is bad, as follows:

(a) State the condition for the voter to reelect the incumbent as a function of \( \mu(a) \), the voter’s posterior belief that the challenger is good as a function of the incumbent’s action \( a \).

(b) Derive the probability that an incumbent who knows the challenger to be bad wins in equilibrium (i.e., if the incumbent advertises).

(c) Derive the probability that an incumbent who knows the challenger to be bad wins off the equilibrium path (i.e., if the incumbent does not advertise).

(d) Derive the condition such that there is a separating equilibrium in which the incumbent advertises if she knows that the challenger is bad.

4. (7.5 points—Miscellaneous) If you have extra time, you may occupy yourself answering the following questions. Please note that each question is worth only 2.5 points—you probably do not want to turn attention to this problem unless you are confident about your answers to earlier problems.

(a) (2.5 points) In problem 1, for the models you listed in part (a), further distinguish among those that assume sincere and strategic voting, respectively. For those models that assume strategic voting, *briefly* explain the role that this assumption plays in the theoretical insights generated by the model.

(b) (2.5 points) In problem 2, assume that the political environment is that in part (c) of the problem, but now assume that there are three political parties, each of which is office seeking. Is there is a Nash equilibrium of this game? If so, what tax rate(s) is (are) chosen in equilibrium? If not, why not?

(c) (2.5 points) In problem 3, how, if at all, would the incumbent’s incentives change if the challenger could also advertise the incumbent’s type? Explain your answer.